

# On $\tau$ -Compactness of Products of $\tau$ -Measurable Operators

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## Abstract

© 2017 Springer Science+Business Media New York Let  $(\mathcal{M}, \tau)$  be a von Neumann algebra of operators on a Hilbert space  $H$ ,  $\tau$  be a faithful normal semifinite trace on  $\mathcal{M}$ . We obtain some new inequalities for rearrangements of  $\tau$ -measurable operators products. We also establish some sufficient  $\tau$ -compactness conditions for products of selfadjoint  $\tau$ -measurable operators. Next we obtain a  $\tau$ -compactness criterion for product of a nonnegative  $\tau$ -measurable operator with an arbitrary  $\tau$ -measurable operator. We construct an example that shows importance of nonnegativity for one of the factors. The similar results are obtained also for elementary operators from  $\mathcal{M}$ . We apply our results to symmetric spaces on  $\mathcal{M}$ . The results are new even for the  $*$ -algebra of all linear bounded operators on  $H$  endowed with the canonical trace  $\tau = \text{tr}$ .

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## Keywords

Elementary operator, Hilbert space, Integrable operator, Linear operator, Normal semifinite trace, Rearrangement, Von Neumann algebra,  $\tau$ -compact operator,  $\tau$ -measurable operator